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is a short distance north of the "milk-dipper" group in that constellation.

Neptune passed opposition with the Sun toward the end of December, and is therefore above the horizon during most of the night. It is in the western part of *Gemini*, and moves about 1° westward during the two months.

I.—A PROPOSED METHOD FOR THE MEASUREMENT AND REDUCTION OF SPECTROGRAMS FOR THE DETERMINATION OF THE RADIAL VELOCITIES OF CELESTIAL OBJECTS.

II.—APPLICATION TO A STUDY OF THE VARIABLE STAR *W SAGITTARII*.*

INTRODUCTION.

Beginning thirty-five years ago, when HUGGINS and VOGEL first made application of DOPPLER'S principle to the determination of radial velocities of stars, the methods employed in this work have been developed until at present an efficient and practically uniform system has been adopted in the various astrophysical observatories. In 1888, when the attempt was made at Potsdam to record photographically the displacement of lines in the stellar spectra, the problem was placed on a firm basis. Subsequently, through improvement in the construction of spectrographs, a more accurate knowledge of the wave-lengths of spectral lines, and signal advances in the methods of measurement and reduction of the spectrograms, the results have attained their present degree of accuracy. The probable error of 22^{km} per second for an average case of velocity determination with the older methods has been reduced to a few tenths of a kilometer at the present time.

Flexure and temperature change during exposure may be said to play no part in the performance of the modern properly equipped spectrograph, and other observational sources of error are well under control. Undoubtedly, the spectrograms of to-day contain information regarding the velocities of celestial light-sources more precise than our present methods of measurement and reduction are capable of bringing out. In these methods there exist certain recognized sources of accidental and systematic error which must be eliminated if the coveted hundredth of a kilometer can ever have any significance.

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* Dissertation in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the University of California. (Abstract. Complete paper published as *L. O. Bulletin*, Number 62.)

PART I.

METHODS OF MEASUREMENT AND REDUCTION OF SPECTROGRAMS.

The plan of procedure that has been widely adopted, subject to slight variation in the case of different observers, may be briefly described. The spectrogram is measured twice, direct and reversed, on an engine supplied with a microscope of adjustable magnifying power, rigidly mounted over a plate-carriage which is moved by delicate screw and micrometer system in one coordinate only, in the direction of extension of the spectrum. A set of measures includes the list of micrometer readings which correspond to the positions of the plate-carriage when the various spectral lines, both bright and dark, occupy in turn a given position with reference to some fixed point in the focal plane of the microscope. These readings, together with the laboratory determinations of the wave-lengths of the spectral lines employed for the measurements, constitute the data necessary for the determination of the radial velocity of the source emitting the light, in terms of the accepted standards of velocity. From the micrometer readings and wave-lengths corresponding to not less than three known lines in the comparison or solar spectrum, the constants of the Hartmann-Cornu interpolation formula* are easily computed. By this formula micrometer readings are readily transformed into approximate wave-lengths, and *vice versa*. If, now, each plate is to be reduced independently, as at Yerkes Observatory,† the constants of the empirical interpolation curve are computed as above from three or more comparison lines. Corrections to this curve are determined by comparison of the adopted and interpolated wave-lengths of the comparison lines, and

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***Astrophysical Journal*, Vol. VIII, page 218. *Astronomische Nachrichten*, Nr. 3702.

† *Publications of the University of Chicago*, Vol. VIII, page 150. *Astrophysical Journal*, Vol. XV, No. 1.

are plotted as functions of the micrometer readings or the λ's themselves. It then remains to compute by means of this corrected interpolation formula the wave-lengths corresponding to the micrometer settings on the lines of the spectrum to be measured. We thus obtain the actual wave-lengths of these lines as affected by the motion of their source relatively to the observer. From the known wave-lengths of these lines, occurring in the spectrum of a light-source at rest with reference to the observer, a simple application of DOPPLER'S principle will yield the desired radial velocity of the star. If, on the other hand, the dispersions of one or more solar plates* be taken as standards, by measures on the comparison lines of a stellar spectrogram, the micrometer readings on the star lines themselves are reduced to the selected dispersion, and may be compared directly with the computed readings of the same lines in the fundamental table. In point of accuracy, there seems to be no choice between these two methods, but the latter one is undoubtedly shorter. The sources of error which are shared by both, I shall briefly review.

(*a*) *Change in film.*—The actual shape of the photographic film while the plate is being exposed is never reproduced, owing to distortions ascribable to temperature change and the processes of development and drying, but the effect of these distortions on velocity determinations is minute and intangible, especially when many well-distributed comparison lines are employed.

(*b*) *The measuring-engine.*—(1) Errors in screw.—Errors due to inequalities in the pitch of the micrometer screw, whether periodic or otherwise, are usually negligible in modern engines. They may, of course, be eliminated by application of small corrections to the measures, but are essentially inoperative in most velocity work.

(2) Temperature change during measures.—The effect of temperature change on the measuring-engine during measurements may play an important part unless considerable care is exercised. Measures should not be begun until the micrometer-head has been in contact with the hand for a short time, and during the measures the room temperature should be kept nearly constant. In my own work, after completing a plate,

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***Astronomische Nachrichten*, Nr. 3702. *Astrophysical Journal*, Vol. VIII, page 124.

I have tested the temperature shift by setting back on the lines first measured, and have invariably found some discrepancy. In one solar plate in particular, which had required considerable time for measurement, this shift amounted to about $.003^{mm}$, or the equivalent of seven kilometers' velocity. This led me to test the character of the shift by remeasurement of some of the remaining comparison lines on the plate. The plot of differences thus obtained approximated a straight line which gradually approached the original curve. Evidently the temperature change was progressive, and was entirely taken up in the first curve. Here, again, the advantage of an even distribution of star and comparison lines is apparent. Evidently, also, the "smoothing out" of curves of residuals from interpolation formulæ should not be practiced without due caution, for temperature change during the measures may produce very appreciable distortions in such a plot.

(*c*) *Subjective errors.*—Discrepancies arising from accidental errors of setting and from personal equation are among the most important to be considered. The former can be reduced only by long practice and the employment of a greater number of lines; the latter have been largely eliminated by the reversal of the spectrogram on the engine,* but it should be noted that in the reversal of a plate the spectrum is simultaneously inverted. Thus, in general, it would hardly be said that the effect of personal equation is entirely compensating in the two measures, for the appearance of a line after inversion might be so changed as seriously to interfere with the duplication of the conditions which existed before reversal. Further, in case of a straight-slit spectrograph the curvature† of the spectral lines would seriously interfere with the elimination of personal equation in this way.

(*d*) *Errors due to the assumption of wave-lengths.*—(1) An instrumental defect.—A source of error by no means inappreciable is found in the relative displacement of comparison and absorption lines due to imperfect focal conditions which obtain outside of given narrow regions of the spectrum, corresponding to the rays which pass through the prism train at

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**L. O. Bulletin*, No. 15, and *Astrophysical Journal*, Vol. XV, page 208.

† Curved slits are used on the Mills spectograph. They give straight spectral lines, which is a decided advantage.

minimum deviation, or corresponding to the intersections of the focal curve with the plane surface of the photographic plate. The so-called wings, which seriously impair the definition of the outlying lines of the comparison spectrum, undoubtedly persist far into the region of apparently sharp focus, giving rise to errors in the measured positions of the bright lines. The corresponding displacements of the absorption lines of the stellar spectra, due to the same cause, are certainly much smaller, if at all appreciable. This difference is due to the fact that much light is concentrated in the bright lines, many of which are strongly exposed in order to bring out the fainter features of the comparison. Thus the wings, though they share a relatively small part of the light, may become quite as dense as the sharp part of the lines, for the density of the sharp part of the lines is but slightly affected by light after a certain time, while the wings continue to spread.

(2) Physical differences.—Aside from subjective errors of measurement, there is no greater source of uncertainty in the reduction of spectrograms than that which arises from the assumption of wave-lengths for the star and comparison lines from ROWLAND'S solar tables. It is doubtful if the conditions of density and internal motion in the Sun and spark are sufficiently alike to warrant this procedure. But until better determinations of spark wave-lengths are available, there seems to be no alternative. A careful discussion of the matter by FROST and ADAMS* has led to the conclusion that "In the present state of our knowledge we . . . cannot say with any certainty how much our results are affected by the use of solar wave-lengths for our Ti lines; but presumably by an amount corresponding to less than 0.02 tenth-meters, or about 1.4 kilometers, and perhaps very much less." Wave-lengths in star and Sun may differ by small amounts, due to pressure, etc., but it seems practicable to assume agreement in the case of solar-type stars. In other cases the chances seem favorable.

(3) Errors in ROWLAND'S table.—ROWLAND'S determinations of solar wave-lengths are probably relatively accurate to the nearest hundredth of an Ångström Unit. Their absolute value need be known only approximately for velocity work.

(4) Errors in practice.—In actual performance some ap-

**Publications of the University of Chicago*, Vol. VIII, page 155.

parently good lines are found to yield consistently poor results, which usually lead to their final rejection. Errors in wave-lengths or intensities in ROWLAND's table, or close lines in the star, may be responsible for the trouble. However that may be, there are undoubtedly among the lines retained many which are similarly in error, but in a smaller degree. It simply remains to eliminate such lines as soon as they can be detected, though the uncertainty of proceeding in the dark in this regard is far from satisfactory.

(*e*) *Elimination of these errors.*—For errors due to change of film and inaccuracies in the measuring-engine, including temperature change during measures, adequate remedy has been proposed, but no elimination of the remaining uncertainties incident to present methods seems possible without a radical change in the system employed. As long as the wave-lengths determined from measures with the grating or interferometer by an observer of one personal equation are employed by another observer of a different personal equation to represent his measures of corresponding lines as produced by a different spectroscope, it can hardly be expected that the results of various observers will be entirely consistent. The problem demands that the conditions which obtain in the production, measurement, and reduction of a spectrographic plate of a star should be exactly duplicated in the production, measurement, and in general the reduction, of the plate from which the fundamental data for velocity determinations are secured. The same spectrograph should produce both plates. They should be measured by the same observer with the same measuring-engine. And, finally, they should be reduced in parallel. These requirements are clearly most exacting, but as a suggestion of a means toward this end, I propose the following method, which, as far as I know, has never been applied in this way.

A PROPOSED METHOD FOR THE MEASUREMENT AND REDUCTION OF SPECTROGRAMS.

Proceeding as in case of a stellar exposure with the spectrograph in final adjustment, from which it must not be disturbed, a source of accurately known velocity, such as the sky or Sun, is photographed with the comparison in the usual manner. Then the measures upon the comparison and continuous spectra of this plate with some engine by any observer will constitute his fundamental standard table for the engine and spectrograph used. These measures of the bright and dark lines of a plate of the sky or Sun and spark will fix the relative positions of the Fraunhofer and comparison lines for a known velocity, and will constitute a velocity-standard table. For the determination of the velocity of any celestial object of the same characteristics as the standard source, it is only necessary to duplicate the exposure and measures of the standard plate and compare directly the relative positions of corresponding lines of the two sources referred to the comparison spectrum of their respective plates. In the reduction of a stellar plate for velocity determination, we first reduce the star measures to the dispersion of the standard plate by forming a simple plot with micrometer readings as abscissæ and differences in settings on comparison lines as ordinates. For the star lines the reductions to fundamental dispersion are read directly from this plot. If these reductions are applied to the readings on the stellar spectrum, the measured positions of any dark line on the Sun and star plates will differ by an amount proportional to the relative velocity toward the Earth of the star and Sun.

The assumptions in this method are fundamental in every case of velocity determination. Further assumptions treated above as incident to other methods are here eliminated by the comparison of artificial lines with artificial lines and dark lines with dark lines. The advantages of this simple system need only be mentioned. Errors in the screw of the measuring-engine are eliminated for small displacements, such as are found in stellar spectra, if the standard and star plates occupy identical positions on the plate-carriage during measures. Personal equation is well controlled without the reversal of the spectrogram, and may be closely followed by frequent measures of standard plates. It may even seem preferable not to reverse the plate, for, as I have suggested, the curvature of lines may introduce a variation of this unaccountable source of error. The magnitude of accidental errors of setting is not affected by the use of this method, but with the greater number of lines available in solar-type stars particularly, the reliability of the means from any plate is clearly increased. Uncertainties due to imperfect focal conditions are not only largely eliminated if the exposures on comparison and star are consistent in star and standard plates, but the region of spectrum available for measurement can be extended without fear of encountering difficulty. The number of lines available is further augmented for solar-type stars by the possibility of including blends, which are clearly reliable under this method. Until tests are made with high-dispersion instruments, it is difficult to predict the extent of the advantage of this increase of the measurable lines for this case; but for low-dispersion instruments in my own experience with a magnifying power of twenty-five, the number of available lines increased from twenty to one hundred and seventy; and with a power of ten, though practically no lines could be used with methods depending on wave-lengths, sixty-five lines gave good results for velocity determinations. But for the present the chief advantage of the method lies in its independence of accurate relative or absolute values of the wave-lengths or intensities of the spectral lines of the spark or continuous spectrum. For the determination of the factors necessary to convert micrometer displacements into kilometers, rough relative values of the wave-lengths of three favorable spark lines are needed, but, aside from this, there is no necessity for data of this kind. It is possible to make final definitive measures at once.

In the application of this method any degree of refinement can be introduced at the will of the observer. The number of plates employed to form a standard table can be increased until the full possibilities of the spectrograph and measuring engine are realized. By comparison of standard plates any errors due to temperature change in the instrument can be detected, and tables can be prepared for different temperatures, if desired. Standard plates can be prepared occasionally and measured, along with the regular observing list, as checks on the constancy of the results. For absolute checks of the performance of the method lunar and planetary spectrograms are of course available, but the determination of the sky or solar velocities furnish equally reliable checks, and at the same time afford data for strengthening the original standard tables. In general, three sky or Sun plates should furnish sufficient data for a fundamental table, though the conditions obtaining in any particular case might call for a greater or smaller number.

This velocity-standard method is best applicable to solar-type stars, but it may, if desired, be modified to suit the requirements of hydrogen or other stars without the introduction of errors greater than those incident to present methods. Thus a standard table for any class of stars may be prepared by introducing in the spark, in addition to the regular comparison, those elements which produce good lines in the stellar spectra. The assumption is made that the relative positions of the spark and dark lines of the elements used are not affected differently either by the dissimilar physical conditions in the spark and star or by the peculiarities of the spectrograph; but this is an assumption commonly made in all spectrographic work.

PART II.

AN APPLICATION OF THE VELOCITY-STANDARD METHOD.

(*a*) *Apparatus employed.*—Necessity for some departure from the present methods of measurement and reduction of spectrograms first arose in the attempt to improve the accuracy of relative velocity determinations from spectrum plates made with *Spectrograph I* of the Lick Observatory. The performance of this spectrograph is exceptionally good. The limit of resolving power with lantern-slide plates is about 0.50 Å.U. at Hγ, which is all that could be expected of an instrument of one fifth the dispersion of the Mills. However, with rapid emulsions, the increased size of silver grains interferes seriously with the definition. Careful tests by Dr. J. STEBBINS and Dr. J. H. MOORE have shown that all effects of flexure have been successfully eliminated in the construction of this instrument. But the effect of temperature change on the positions of lines is very marked. This is largely due to the unequal expansion in the triangle formed by the steel collimator tube, the brass camera tube, and the brass tie-rods which extend from the collimator tube nearly to the plate-holder. The extent of this shift has been determined by Dr. MOORE. It is equivalent to approximately 36 kilometers in velocity per degree Centigrade change in temperature. The limitations of the instrument are thus readily appreciated. In my own work, in the absence of a temperature case, the spectrograph has been wrapped in several thicknesses of woolen blanket just before the exposure began. At the same time all the windows in the dome were closed. These precautions, together with the frequent introduction of the comparison, have made good results possible, but it has been almost invariably the case that the best velocity determinations have been made from plates for which the temperature range was a minimum. Recently a temperature case and automatic thermostat have been constructed for this instrument. These may be expected to result in very material improvement in its performance.

Further disadvantages are inherent in an instrument of low dispersion and resolving power. Linear defects in lines produce five times the error that they would occasion with the Mills spectrograph. Probably no more than fifteen or twenty single lines could be measured with any power on any one plate of a solar-type star. STEBBINS found only six coincidences of solar and stellar lines for *o Ceti* in a region of 340 Å.U., while in a region covering 120 Å.U. he found twenty such lines on Mills plates. With a power of ten, absolutely no single lines would be available for velocity determinations. In view of these facts, it would seem that few tests could be more exacting than the application of the velocity-standard method to this case. Spectrograms made with an instrument of this kind, with no greater protection against temperature change, are affected by errors which accurate measurements serve to bring out; but it was one aim of the author to determine the degree of accuracy to be expected from an instrument of low dispersion, for its field of usefulness is practically inexhaustible, as it reduces the exposure time required with the Mills spectrograph by ninety per cent.

(*b*) *Application to the study of the variable star, W Sagittarii.*—Exhaustive studies of the light curve of *W Sagittarii* were made by Herr J. H. F. SCHMIDT, who discovered the variability of this star, then known as γ' *Sagittarii,* at Athens in 1866.* From a series of eight hundred and ninety observations, covering ten years and extending over one hundred and ninety-five maxima and one hundred and ninety-three minima, he constructed a light curve, which is drawn in detail in Plate

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* *Astronomische Nachrichten*, Nr. 2071, page 103.

8* of this paper. In addition to the strong irregularities in the curve, he suspected a perturbation in the light period running through a cycle of eight years, affecting the time of maximum and minimum several tenths of a day. But his observations were hardly extensive enough to warrant this last conclusion. The spectrographic observations at present available do not cover a period long enough to confirm or to disprove this result. As presented in the Chandler and Harvard catalogues, the important data regarding this star, with some additions, are as follows:—

Chandler's Third Catalogue.

Max., $4^{m}.8$; Min., $5^{m}.8$; M-m, $3^{d}.00$; Period, $7^{d}.59460E$.
Epoch of Max., 1866, Sept. 4; Julian, $2{,}402{,}849^{d}.45$.

Harvard Catalogues.

Max., $4^{m}.3$; Min., $5^{m}.1$; Class IV; Sp., G5K.
α 1900.0, $17^{h}\ 58^{m}.6$; δ 1900.0, $-29^{\circ}\ 35'$.

Additional Data.

λ 1902.0, $269^{\circ}\ 44'$; β 1902.0, $-6^{\circ}\ 08'$.
Photo. Max., $5^{m}.5$; Photo. Min., $6^{m}.5$.

The spectrum of *W Sagittarii* approximates very closely to the solar type. The fact that sixty-five blends ranging in intensity from twelve to forty gave good results, as exhibited in the last column of Table III,† establishes the close resemblance of this star to our own central body. The seventeen lines in Table IV† suggest some differences, the most striking of which is the strong line at λ 4335.7, which is not found in the Sun, but rises to an intensity of 12 in the star. But, on the whole, the identities are far-reaching enough to warrant an extensive system of blending.

This fact justified the use of a power of ten in the measures of the plates of *W Sagittarii*. But the selection of such a low power was not decided upon without considerable experiment with a higher one. With a power of ten, the width of the micrometer thread (about 0.50 Å.U.) interfered with settings on lines of intensity 12 to 15, while the density of the comparison lines was so great that it became difficult at times to distinguish the black vertical wire when set on them. After experimenting with powers of ten, twelve, fifteen, and twenty-five, I am inclined to think that a power in the neighborhood of fifteen would perform most satisfactorily in the case of solar-type stars.

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* Omitted in this abstract.

† Tables and details of reduction are omitted in this abstract.

From preliminary measures in August, 1903, of three spectrograms of *W Sagittarii,* a very considerable variation in the radial velocity of this star was brought to light. Subsequently a series of thirty-three spectrograms, distributed uniformly in the light period, was completed with Spectrograph I. These have been measured and reduced for the purpose of determining the character of the orbit of this interesting variable. I have not been able to recognize upon these plates any effect attributable to the light of the fainter companion now known to exist, but it should be remembered that a difference of one magnitude is nearly sufficient to obliterate the effect of the darker star.

Assuming the identity of the light and velocity periods for this star, we may lay down the plot of the velocity determinations by employing the intervals after maximum as abscissæ and the corresponding radial velocities of the star with respect to the Sun as ordinates. Ample verification of the adopted value for the period was found in the close agreement between results obtained from plates covering more than forty maxima of the curve, or an interval of 309 days. But when the attempt was made to pass an elliptic velocity curve through these observations, it was found that the plotted points oscillated above and below this curve with a period of 3.8 days, or one half that of the light variation. After repeated trials of various ellipses with different values of periastron time, longitude of periastron, eccentricity, maximum positive and negative velocity, and the velocity of the system, I selected the velocity curve which is drawn on Plate 8* with the narrower line. In selecting this conic it was assumed that the actual observed velocities followed a superimposed curve with a period of 3.8 days and with nearly equal amplitudes for crests and troughs. For the better determination of the secondary curve the residuals of all plates from the ellipse were plotted after being reduced to one complete period of 3.8 days, employing the wellestablished nodal point at 1.7 days after the light maximum. A sine curve was then passed through these points with an amplitude of 4.2^{km} at the crest and 5.5^{km} at the trough. This final curve was superimposed on the velocity curve and is represented by the heavy line of the upper curve of Plate 8.* The residuals from this curve for all the plates appear in Table VII* under the heading *r*. Including all these residuals, and assigning equal weights to each, the resulting probable error of a single plate is $\pm 0.90^{\text{km}}$. Excluding five inferior plates whose velocities could be given very small weight in the construction of the curve, the probable error of any one of the remaining twenty-eight plates is $\pm 0.55^{\text{km}}$. The occurrence of relatively larger residuals along the crest of the secondary curve finds ready explanation when it is noticed that, of the two crests which obtain in one complete revolution of the system, the one occurs at the light maximum where the greater activity of the star could lead to wide ranges in velocity, while the other occurs at light minimum where, with increased exposure time, the effect of temperature change in the instrument becomes a maximum.

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* Omitted in this abstract.

The elements of the orbital or primary curve of the brighter component of *W Sagittarii,* together with those of the superimposed secondary curve, as are follows:—

	ELEMENTS	Primary Curve.	Secondary Curve.
Apparent period,	P'	7.59460 days	3.80 days
Longitude of periastron,	ω	$70^\circ.0$	
Eccentricity,	e	0.320	0.0
Time of periastron after light maximum,	T	6.20 days	
Projection of semi-major axis on plane of sight,	$a \sin i$	$1{,}930{,}000^{\text{km}}$	
Projection of periastron distance on plane of sight,	$q \sin i$	$1{,}310{,}000^{\text{km}}$	
Projection of apastron distance on plane of sight,	$q' \sin i$	$2{,}550{,}000^{\text{km}}$	
Ratio of masses,	$\dfrac{m^3 \sin^3 i}{(m + m_1)^2}$	$0.00499 \odot$	
Amplitude of velocity curve at crest,	A	$+21.6^{\text{km}}$	$+4.2^{\text{km}}$
Amplitude of velocity curve at trough,	B	-17.4^{km}	-5.5^{km}
Velocity of center of mass of the system,	V	-28.6^{km}	

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* Omitted in this abstract.

Our present knowledge of the elements of *W Sagittarii* makes possible the introduction of two small corrections to the period of this star depending upon the velocity of light. The first is due to the change in the distance between the Earth and the bright component of the binary system occasioned by their orbital motions, and may be regarded as the combined effect of the equations of light of the two bodies. Its operation is not systematic, but may be taken into account in the computed times for maxima used in plotting the velocity observations by changes never exceeding 0.01 days. The maximum variation from the mean in the present problem is about 0.002 days, which is equivalent to a displacement of 0.007^{cm} in the abscissæ of my original velocity diagram. It is therefore evident that this correction is negligible. The second correction to the period of this variable is based upon the continued approach of the system toward the Sun at the rate of 28.6^{km} per second. As this correction is constant, the use of the apparent period for the velocity diagram does not affect the determination of the elements which characterize the form and position of the orbit. But the epochs of the curve, referred to the time of maximum, and all dimensions of the orbit are direct functions of the true period. The correction to the apparent period amounts to + 62 seconds, or + 0.00072 days. Accordingly, the true value of the light and velocity periods of *W Sagittarii* is—

$$P = 7.59532 \text{ days.}$$

As the remaining elements of the velocity curve are published to three places only, this change in the fifth significant figure of P does not appreciably affect their values.

The angle i, which measures the inclination of the orbital plane to a plane tangent to the celestial sphere at the center of mass of the system, cannot be determined without micrometrical measures of the two components. The absolute scale of the ellipse is therefore indeterminate.

For purposes of comparison among spectroscopic studies of *Cepheid* variables, there exist in addition to the present investigation the excellent results of WRIGHT and BÉLOPOLSKY in connection with their researches regarding the orbits of δ *Cephei** and of η *Aquilæ*.† A strong resemblance between the elements of these stars is at once evident. All show a pronounced eccentricity. Also, the time of closest approach of the bodies of the system occurs about a day before maximum brightness. The most marked analogies exist between WRIGHT's elements of *η Aquilæ* and those of *W Sagittarii;* indeed, with the exception of a small difference in eccentricity, the uncertainty of inclination, and a difference of 0.5 days in the period, the two orbits are essentially identical even in regard to the positions of principal maximum and minimum. It is interesting to notice that Mr. WRIGHT has obtained no evidence of a secondary curve for *η Aquilæ,* but such a curve would not have been recognized if its amplitude did not exceed 2^{km}, and in some positions no great distortion of the ellipse would occur with an amplitude of five kilometers for the superimposed curve. Referring to Mr. WRIGHT's article, it should be noticed that the velocity curve reproduced there is the empirical curve which was drawn for the purpose of determining the elements. His actual ellipse passes slightly below observation 11, not above it. It will also be noticed that his observations tend above the published curve from 1 day to 2.8 days after maximum and fall below from 2.8 to 4.5 days. It therefore seems probable that some curve oscillating about an ellipse would represent the observations better.

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* *Astrophysical Journal*, Vol. I, page 160.

† *Astrophysical Journal*, Vol. VI, page 393. Ibid., Vol. IX, page 59.

Mr. WRIGHT's observations extend over four months, and my own over a period of less than a year. It is therefore probable that any influence connected with the long-period irregularities in the light recurrence of these stars would escape detection. Further study of these stars should reveal such phenomena.

The observed uniform correspondence between the light variations and the orbital conditions of these three stars suggests the mutual dependence of the two phenomena. Whatever may be the cause or causes of variation in the *Geminids,* they may not operate in the same manner in the *Cepheids.* It therefore seems advisable to treat each group separately in any attempt to construct a theory for the explanation of their light changes. I shall therefore advance some data regarding the most probable conditions that obtain in the system of *W Sagittarii.*

DARWIN'S expression for tidal potential is—

$$V = \frac{3}{2}\frac{m}{r^3}\rho^2 (\cos^2 z - 1/3),$$

where r is the distance between the masses, ρ is the radius of the disturbed body, and z the angle between r and ρ; m is the mass of the disturbing body. Confining our attention to the line joining the two bodies, we place $z = 0°$ and differentiate with reference to the direction of displacement and arrive at the expression, $F = 2m\rho/r^3$, where F is the tidal force. As the ratio of apastron to periastron distance in this system is 2 to 1, it is evident that the tidal force varies in the ratio of 1 to 8 for these two positions. Using the rough conclusions as to the most probable data for the system arrived at above, we find that the intensity of the tidal force acting on the brighter body is, roughly, 50,000 times that of the Moon acting on the Earth. Further, by the introduction of another term usually negligible in tidal computations, we find the ratio of the intensities of the tidal forces on the side toward the disturbing body to the corresponding forces on the opposite side of the brighter star is 15 to 10 at periastron and 13 to 10 at apastron.

While the increase of light may be due to enormous tidal disruptions in the molten matter of the star's surface, accompanied perhaps with a liberation of heat from below, it should be noted that the displacements of the absorption lines in the spectrum are caused by the motions of the star's atmosphere with and relative to the regions emitting the light. In the atmospheric regions above the high tidal area there would undoubtedly be an uprush of gases due to atmospheric tides, convection currents, increased light pressure, and explosive outburst. Again, in the belt of low tide ninety degrees from the two high tides, there would probably be a corresponding recession of the atmosphere toward the star's surface. This would clearly give rise, in the actual velocity curve, to an oscillation with respect to a true ellipse. These oscillations would fall below the mean at conjunction and above the mean at elongation, for at conjunction the high tidal areas are presented to the observer, and at elongation the low tidal areas. Consulting the diagrams, this is found to be the case. We can therefore depart from the rotation theory to account for the secondary curve, and indeed this seems justifiable, for it is possible with the operation of such immense tides through long periods of time that the rotation periods of the stars involved should be brought to identity with the revolution period, as in case of our own satellite. Further, on the basis of the tidal theory, there would not seem to be sufficient variation in brightness due to difference in intensity of the tidal force between the two hemispheres (the ratio is about 7/5) to produce by rotation the oscillation observed, unless we assume much greater masses for the system. The occurrence of the light maximum when the tidal forces have fallen one third in intensity presents an anomaly, but it is possible that the effect of these forces lags to this extent.

Returning again to the rotation interpretation, it is easy to construct a plausible explanation for the light and velocity curves of *W Sagittarii* on the assumption that the system is pervaded by a resisting medium which enhances the brightness of that side of the star which faces the direction of motion. Or, again, a third body might be present in the system and give rise to the perturbations observed by SCHMIDT in the period. Until more data are available, to follow out such theories would be premature.

Considering all evidence, it seems reasonably certain that the star's variations in brightness, and particularly the principal variations, are attributable to the action of external forces.

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